The Influence of Antenna Pattern on Faraday Rotation in Remote Sensing at L-Band

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***Abstract*—The influence of the pattern of the receive antenna on measured Faraday rotation is examined in the context of passive remote sensing of soil moisture and ocean salinity at L-band. Faraday rotation is an important consideration for radiometers on future missions in space, such as SMOS and Aquarius. Using the radiometer on Aquarius as an example, it is shown that, while I = Tv + Th is independent of Faraday rotation to first order, it has rotation dependence when realistic antenna patterns are included in the analysis. In addition, it is shown that using the third Stokes parameter to measure the rotation angle can yield a result that is biased by as much as 1° by purely geometrical issues that are associated with the finite width of the main beam.**

***Index Terms*—Antenna patterns, Faraday rotation, microwave remote sensing, radiometer.**

I. Introduction

FARADAY rotation is a change in the polarization vector that occurs as electromagnetic waves propagate through the ionosphere. The magnitude of the change varies as $1/(\text{frequency})^2$ and is an important consideration for remote sensing at the low-frequency end of the microwave spectrum. For example, at L-band (1.4 GHz), where remote sensing of soil moisture and sea surface salinity is performed, the rotation of the polarization vector can range from a few degrees to more than 15°, depending on viewing angle and the solar cycle [1]. The corresponding change in apparent brightness temperature can be several kelvin and is an important issue for missions such as SMOS (Soil Moisture and Ocean Salinity) [2], [3] and Aquarius [4], [5] which will be launched soon to measure soil moisture and sea surface salinity at L-band.

Unfortunately, current models for the ionosphere are often not sufficiently accurate to make corrections [6]. This is particularly true in the case of sea surface salinity, which requires high accuracy and measurements over the oceans [4], [7] where data on the ionosphere is sparse.

Among the strategies that are adopted to avoid the changes due to Faraday rotation is to use the first Stokes parameter I = Tv + Th. In the ideal case when the antenna patterns for the two polarizations are identical and there is no cross-polarization coupling, I is independent of Faraday rotation. It is also independent of other rotations such as errors in the antenna polarization clocking angle (mechanical misalignment in the plane perpendicular to antenna boresight that causes the axes corresponding to H- and V-polarization to be rotated relative to their desired position).

Another strategy is to measure the third Stokes parameter TU [see (5) for a definition]. One can show that the ratio of TU to the second Stokes parameter Q = Tv − Th is proportional to the tangent of twice the angle of Faraday rotation. This was recognized by Yueh, who described how TU and Q could be used to measure the Faraday rotation [8]. The use of TU to measure Faraday rotation has been described in the context of SMOS [9], and an analysis of the impact of Faraday rotation on the measurement of TU by Windsat has been described [10].

Both of these strategies work in the case of narrow beam antennas with no cross-polarization coupling. However, at L-band, antennas in space tend to have large footprints (e.g., 100-km diameter for Aquarius) and small but not negligible cross-polarization coupling. The purpose of this paper is to examine the performance of these two approaches when used with antennas with realistic patterns. Of concern are the effect of cross-polarization coupling, the mismatch of the patterns for the two polarizations, and the effect of changes in the orientation of the polarization vectors at the surface with respect to boresight over the footprint of the antenna beam. For example, cross-polarization coupling can introduce a dependence on Faraday rotation in the sum I = Tv + Th and also introduce a bias in the estimate of the angle of Faraday rotation that is obtained from Q and TU. In the sections to follow, expressions for I, Q, and TU are derived for a general antenna and examined in special cases. Then, to get realistic estimates of the magnitude of the effects to be expected in the general case, the patterns of the antennas for the Aquarius radiometer are used to generate numerical results.

II. Antenna Temperature: General Case

Consider a dual-polarized antenna with its two polarization ports $\mathbf{v}$ and $\mathbf{h}$ arranged, so that at boresight, the directions correspond to the conventional definitions at the surface, i.e.,

$$\begin{aligned} \mathbf{h} &= (\mathbf{k} \times \mathbf{n})/|\mathbf{k} \times \mathbf{n}| \\ \mathbf{v} &= \mathbf{h} \times \mathbf{k} \end{aligned} \tag{1}$$

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where $\mathbf{n}$ is a vector normal to the surface, and $\mathbf{k}$ is the direction of propagation from the surface toward the antenna. Let the antenna "voltage" pattern at each port be

$$\begin{aligned} \mathrm{G}_h &= \mathrm{g_{hh}}\boldsymbol{\varepsilon_2} + \mathrm{g_{hv}}\boldsymbol{\varepsilon_1} \\ \mathrm{G}_v &= \mathrm{g_{vv}}\boldsymbol{\varepsilon_1} + \mathrm{g_{vh}}\boldsymbol{\varepsilon_2} \end{aligned} \tag{2}$$

where g_{ij} are complex, and $\boldsymbol{\varepsilon_i}$ are unit vectors defined by Ludwig [11] to indicate the directions of copolarization and cross polarization. Assume a local coordinate system at the antenna with unit vectors ($\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$), and let the z-axis be along the boresight direction (pointing to the surface) and let the x-axis be aligned with the direction of vertical polarization $\mathbf{v}$ at the surface. Then, one has [11]

$$\begin{aligned} \boldsymbol{\varepsilon_1} &= \left[1+\cos^2\varphi(\cos\theta-1)\right]\mathbf{x} \\ &\quad + (\cos\theta-1)\sin\varphi\cos\varphi\mathbf{y} - \sin\theta\cos\varphi\mathbf{z} \\ \boldsymbol{\varepsilon_2} &= (\cos\theta-1)\sin\varphi\cos\varphi\mathbf{x} \\ &\quad + \left[1+\sin^2\varphi(\cos\theta-1)\right]\mathbf{y} - \sin\theta\sin\varphi\mathbf{z}. \end{aligned} \tag{3}$$

The antenna output, i.e., antenna temperature T_A, can be written in the form [12]

$$\mathrm{T}_A = \int \mathrm{G}(\Omega)\mathrm{R}(\Omega)\mathrm{T}_B(\Omega)\, d\Omega \tag{4}$$

where T_B is the "modified" Stokes vector, in units of brightness temperature, evaluated at the surface

$$\mathrm{T}_B = \begin{bmatrix} \mathrm{Tv} \\ \mathrm{Th} \\ \mathrm{T3} \\ \mathrm{T4} \end{bmatrix} \tag{5}$$

where $\mathrm{T3} = \mathrm{TU} = 2\alpha\mathrm{Re}\langle \mathrm{Eh}^*\mathrm{Ev}\rangle$, and $\mathrm{T4} = \mathrm{TV} = 2\alpha\mathrm{Im}\langle \mathrm{Eh}^*\mathrm{Ev}\rangle$. In these expressions, Ev and Eh are the electric fields of vertical and horizontal polarizations, respectively; the coefficient of proportionality is $\alpha = \lambda^2/(\eta k)$, where $\eta = \sqrt{\boldsymbol{\mu}/\boldsymbol{\varepsilon}}$ is the intrinsic impedance of the medium, k is Boltzmann's constant, and $\langle\rangle$ indicates the expected value.

In (4), R is a "rotation" matrix given by

$$\mathrm{R} = \begin{bmatrix} \cos^2\varphi_c & \sin^2\varphi_c & 0.5\sin 2\varphi_c & 0 \\ \sin^2\varphi_c & \cos^2\varphi_c & -0.5\sin 2\varphi_c & 0 \\ \sin 2\varphi_c & -\sin 2\varphi_c & -\cos 2\varphi_c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{6}$$

and G in (4) is an antenna pattern matrix given by (7), shown at the bottom of the page.

The matrix in (7) appears in scattering theory where the parameters g_{ij} are replaced by scattering coefficients, and it is commonly called the "Stokes matrix" [13], [14].

In (6), the angle $\varphi_c = \varphi + \boldsymbol{\Phi}_F$, where $\boldsymbol{\Phi}_F$ is the Faraday rotation angle (Appendix B-I) and φ is a geometry-dependent rotation. The latter occurs because the polarization vectors that are defined on the surface [$\mathbf{h}$ and $\mathbf{v}$ in (1)] are aligned with the polarization vectors of the antenna (3) only at boresight. Along other rays from the surface to the antenna, the polarization vectors at the surface are rotated relative to the vectors that were defined in (3) (see Appendix B-II and also [15]). Although not considered here, it is also possible to have a rotation about boresight of the antenna polarization vectors themselves relative to the desired orientation (e.g., a misalignment of the polarization vectors due to mechanical error). This would appear as a constant offset φ_0 that would be included in φ_c.

III. Special Cases

The matrix operations that result from substituting (5)–(7) into the integrand of (4) are straightforward, but the expressions that result are rather long. The general expressions are given in Appendix A, and they will be used for the numerical computations to be discussed in the following (Section IV). However, in order to gain insight, it is convenient to first look at special cases. In the following discussion, $\mathrm{I} = \mathrm{Tv} + \mathrm{Th}$, $\mathrm{Q} = \mathrm{Tv} - \mathrm{Th}$, and $\mathrm{T3} = 2\alpha\mathrm{Re}\langle\mathrm{Eh}^*\mathrm{Ev}\rangle$. Parameters without primes are measured at the surface. Parameters with primes (i.e., I', Q', and $\mathrm{T3}'$) have the same definition but are measured at the sensor after propagation through the ionosphere and after being weighted by the antenna pattern (7) but before integration. That is, they are the result of the matrix product $\mathrm{G}(\Omega)\mathrm{R}(\Omega)\mathrm{T}_B(\Omega)$ in (4).

A. Ideal Antenna Patterns

The general expressions simplify greatly if one assumes that the antenna patterns for the two polarizations are identical and that there is no cross-polarization coupling. In particular, assume $\mathrm{g_{hh}} = \mathrm{g_{vv}} = \mathrm{G}$, and $\mathrm{g_{hv}} = \mathrm{g_{vh}} = 0$. In this case, one obtains the conventional results, and the first Stokes parameter $\mathrm{I}' = \mathrm{Tv}' + \mathrm{Th}'$ is independent of Faraday rotation. Combining the first two rows in the integrand in (4), one obtains

$$\mathrm{I}' = \mathrm{G}^2\mathrm{I} \tag{8a}$$

$$\mathrm{Q}' = \mathrm{G}^2\cos(2\varphi_c)\mathrm{Q} + \mathrm{G}^2\sin(2\varphi_c)\mathrm{T3} \tag{8b}$$

$$\mathrm{T3}' = \mathrm{G}^2\sin(2\varphi_c)\mathrm{Q} - \mathrm{G}^2\cos(2\varphi_c)\mathrm{T3} \tag{8c}$$

where it has been assumed that $\mathrm{T4} = 0$ at the surface, but $\mathrm{T3} \neq 0$ and the primes on the quantities on the left are a reminder that the integration in (4) has not been done.

$$\mathrm{G} = \begin{bmatrix} |\mathrm{g_{vv}}|^2 & |\mathrm{g_{vh}}|_2 & \mathrm{Re}\,(\mathrm{g_{vv}}\mathrm{g_{vh}^*}) & -\mathrm{Im}\,(\mathrm{g_{vv}}\mathrm{g_{vh}^*}) \\ |\mathrm{g_{hv}}|^2 & |\mathrm{g_{hh}}|^2 & \mathrm{Re}\,(\mathrm{g_{hh}^*}\mathrm{g_{hv}}) & -\mathrm{Im}\,(\mathrm{g_{hh}^*}\mathrm{g_{hv}}) \\ 2\mathrm{Re}\,(\mathrm{g_{vv}}\mathrm{g_{hv}^*}) & 2\mathrm{Re}\,(\mathrm{g_{hh}^*}\mathrm{g_{vh}}) & \mathrm{Re}\,(\mathrm{g_{vv}}\mathrm{g_{hh}^*} + \mathrm{g_{vh}}\mathrm{g_{hv}^*}) & -\mathrm{Im}\,(\mathrm{g_{vv}}\mathrm{g_{hh}^*} - \mathrm{g_{vh}}\mathrm{g_{hv}^*}) \\ 2\mathrm{Im}\,(\mathrm{g_{vv}}\mathrm{g_{hv}^*}) & 2\mathrm{Im}\,(\mathrm{g_{hh}^*}\mathrm{g_{vh}}) & \mathrm{Im}\,(\mathrm{g_{vv}}\mathrm{g_{hh}^*} + \mathrm{g_{vh}}\mathrm{g_{hv}^*}) & \mathrm{Re}\,(\mathrm{g_{vv}}\mathrm{g_{hh}^*} - \mathrm{g_{vh}}\mathrm{g_{hv}^*}) \end{bmatrix} \tag{7}$$